Problem Chosen	2020	Team Control Number
B	MCM/ICM Summary Sheet	2013596

The Longest Lasting Sandcastle(s)

Summary

The warm sunshine, the vast sea and the sand beach are the dream scenery for all of us. When going to the beach, we all like to create sandcastles. Both children and adults can immerse themselves in the interesting game. Compared with the endless sea, the sandcastle seems so small and fragile. How can it be maintained longer?

In response to this question, we build the model from two perspectives. We consider the impact of the waves on the shape of sandcastles from the top and side views. In this way, we convert three-dimensional complex problems into two two-dimensional planar problems so that the model can be simplified to facilitate modeling and simulation. At the same time, the erosion at the model boundary can be analyzed more clearly.

For the top view sandcastle model, it is based on the free version of CFD tool in MATLAB. We simulate sandcastles of the same volume and different shapes, and plot the flow direction and force of the waves after they hit the sandcastle. In the end, beautiful results are carried out.

For the side view sandcastle model, we also model and simulate based on MATLAB. This model reflects the situation when the outer contour is eroded by the impact of the waves.

We collect rich and effective references. We perform force analysis, data fitting, model building, and simulation. Besides, we take advantage of many tools and models such as the CFD toolbox, cellular automata and so on to analyze and solve problems as accurately and comprehensively as possible.

In the problem I, we build two kinds of models and analyze three typical sandcastle shapes, triangular pyramid, hemisphere and water-drop type, by way of example. In the problem II， we are committed to figure out the physical relationship between parameters such as liquid bridge force, water content, and sand diameter. In the problem III, we analyze the various effects of rain on sand. Besides, based on the cellular automata, we build a dynamic simulation model of the rain-eroded sandcastle. At last, we also put forward our own ideas about the strategies to make sandcastle last longer in problem IV.

From our humble point of view, the water-drop type sandcastle is relatively more stable. This result seems to be both unexpected and reasonable. It fits our common sense to some extent. But if rains, the shape of hemisphere shows better resistance to erosion than the water-drop type. Various analyses and conclusions on the stability of sandcastles are expected in more detail later.

In a word, we build a multi-angle, complete mathematical model to analyze the stability of the sandcastle, and provide effective methods to keep sandcastles lasting longer under the waves and rain erosion.

Fun in the Sun--How to Make Sandcastles Stronger

Hello everyone, happy holidays and welcome to the beautiful beach! The beach is one of the first resorts for people when they are on vacation. Many people like to build sandcastles on the beach. Wet sand on the beach instead of dry sand has a strong plasticity. You can use sand to pile any shape of your favorite castle. But because of the beating of the waves and the erosion of the tide, the beautiful sand castle will slowly disappear. How can sand castles be kept longer? We have studied this issue and have come up with some useful conclusions. Just keep reading.

First, we consider which 3-dimensional geometric shape of the sandcastle foundation can withstand longer-term seawater erosion with the same wet sand quality and erosion. From a side perspective, people seem to build more straight bevels because it is easier to build. We compare the speed of sandcastle erosion when the edges, slopes, and arcs are facing the sea. Interestingly, the curved sandcastle has the slowest erosion rate. In other words, compared with the sharp-edged sandcastle, the smooth-shaped sandcastle is a better choice. Then, from a bird's-eye view, we design a sandcastle with a triangular, circular, and drop-shaped bird's eye view. By simulating the erosion of the seawater, we find that the drop-shaped sandcastle is the least stressed. Now you should know that the sandcastle with the drop-shaped side and the arc on the side is the strongest, which is the streamlined sandcastle we usually say. In addition to the erosion of the seawater, rainfall also washed away sandcastles. Similarly, when the castle top is a smoother arc, it is more conducive to the preservation of the sandcastle.

Then we look at the moisture content of the sand, that is, the effect of the degree of wetness on the stability of the sandcastle. We all know that dry sand is too loose, wet sand with too much water flows, and neither can be shaped. Therefore, the proper wetness of the sand has a significant impact on the robustness of the castle. Through analysis, we find that sand with a water content of 18%-24% is most suitable for making sandcastles. The erosion of seawater will make the water content of sand higher and higher, so you can consider drying the sand slightly before building a sand castle. Remember to slap the sand during the construction process to make the sand castle stronger!

If other materials are on hand, they may be useful tools to make sandcastles last longer. For example, adding a baffle in front of a sandcastle can greatly reduce the impact of seawater, adding clay to the sand can make the sandcastle more compact, and more simply, moving the sandcastle away from the seawater can reduce the erosion of the sandcastle by high tide.

The above are some of the methods we have made to make sand castles stronger. To build a streamlined sandcastle, the water content of the sand should not be too much, and tools should be effectively used. Enjoy your vacation and build a beautiful sandcastle!

Contents

1 Introduction

1.1 Background

Wherever there are recreational sandy ocean beaches in the world, there seem to be children (and adults) creating sandcastles on the seashore. Using tools, toys, and imagination, beach goers create sandcastles that range from simple mounds of sand to complicated replicas of actual castles with walls, towers, moats, and other features that mimic real castles. In all these, one typically forms an initial foundation consisting of a single, nondescript mound of wetted sand, and then proceeds to cut and shape this base into a recognizable 3-dimensional geometric shape upon which to build the more castle-defining features.

Inevitably, the inflow of ocean waves coupled with rising tides erodes sandcastles. It appears, however, that not all sandcastles react the same way to waves and tides, even if built roughly the same size and at roughly the same distance from the water on the same beach. Consequently, one wonders if there exists a best 3-dimensional geometric shape to use for a sandcastle foundation.

1.2 Restatement of the Problem

We are required to to solve the following four problems.

The first part is to establish a model to simulate the erosion process of sea water and tide, to solve the optimal three-dimensional geometry of sandcastles which can last the longest, and to work out the longest time.

The second part is to use the model to determine the best sand water mixing ratio of the castle foundation.

The third part is to analyze whether the above strategy is still the best when considering the rain erosion.

The fourth part is to give more plans to extend the life of sandcastles.

2 Assumptions and Notations

2.1 Assumptions

To simplify our problem, we make the following basic assumptions, each of which is reasonable.

Only the mechanical erosion of seawater is considered. Because chemical erosion is caused by the chemical reaction of water containing carbonic acid or organic acid with soluble rock, it takes much longer than mechanical erosion to produce relatively weak effect.

Regardless of other solid substances in the sea water, only the mechanical erosion caused by the impact between the sea water and sand castle is considered.

A line parallel to the coastline has the same magnitude and direction of force. Although the wave shapes differ from crests and troughs, the position and direction of the waves flowing to the sandcastle are not the same. In fact, waves have the same probability to head towards every location on the shore. From this, we can regard the linear position parallel to the coast as the contour line during the wave propulsion. At the same time, the size of the isoline decreases with height and distance to the coast.

2.2 Notations

Abbreviation	Description
E	Amount of sand erosion per point
d	Sandcastle deformation rate
ρ_s	Sand density
ρ_w	Seawater density
V_s	The initial volume of the sand castle
u	Wave velocity
K	Conversion coefficient of erosive force and amount
φ	Internal friction angle of sand
τ_w	Erosion shear stress of waves
τ_c	Shear stress of sand against erosion
R	Sand radius
b	Radius of contact between liquid bridge and particles
σ	Water surface tension

3 Problem I: the Best Sandcastle Shape

3.1 Model Analysis

3.1.1 Model Classification

In terms of modeling and simulation, we consider simplifying the shape changes from two perspectives: top view and side view. We convert three-dimensional complex problems into two two-dimensional planar problems so that the model can be simplified to facilitate modeling and simulation. At the same time, the erosion at the model boundary can be analyzed more clearly.

We assume that the waves are moving towards the beach at a constant speed, and the sea level height increases linearly with time. It is assumed that the initial water content causes the sand to fall immediately after being impacted by seawater, and the water content is constant. When the number of sand particles decreased to 50% of the original, we thought that the sand castle was destroyed. We also assume that all sands are uniform spheres of equal size, and the size and shape of the liquid bridge formed by water between the sands are exactly the same. The completed sandcastle has uniform voids and the same water permeability everywhere.

Top View Sandcastle Model

Based on the free version of CFD tool, which is an extended toolbox of MATLAB, we build the top view hydrodynamic model of sandcastle. In the model, we build the wave and different types of sandcastle shapes. Then, we set the related parameters and simulate the process of sea water washing sandcastles. After that, we analyze and integrate the forces on the surface of sandcastles, and compare the average impact forces of different shapes of sandcastles to get the best sandcastle.

Side View Sandcastle Model

With MATLAB as the platform, we build the side model of wave impacting sandcastles. In the side model, we simulate the different height relationship between waves and tides more carefully. At the same time, surf simulation is carried out on the sides of sand castle with different shapes to calculate the resultant force.

3.1.2 Formulas and Parameters

In terms of analysis and calculation, we perform a physics analysis of the waves washing sandcastles. After consulting some references, we sort out and optimize the following expression formulas and parameters:

Amount of sand erosion per point

First, we figure out the relationship between the stress on the sandcastle and the amount of wave erosion. The calculation method is as follows:

$$\Delta E = K\,(\tau_w - \tau_c)^{a} \tag{1}$$

where ΔE is the amount of sand erosion per point, K and a are conversion index of erosive force and its amount. Referring the related information, we set the value $K = 4.57mm/d$, and $a = 1$.

The τ_w is the erosion shear stress of waves. The τ_c is the shear stress of sand against erosion. It is related to the liquid bridge force inside the sand and the water content of the sand. To simplify, we regard it as a constant in the first question model. In the second question of the model to further explore.

Erosion shear stress of waves
The friction stress of the beach bottom surface is expressed by the wave velocity as follows[1]:

$$\tau_w = \frac{\varepsilon \rho_w l u^2}{2} \tag{2}$$

where ε is the index of friction. The general value range is *0.01~0.10*. In our model, we take the value ε*=0.05*. ρ_w is the seawater density, so $\rho_w = 1.025 g/cm^2$. l is length of fluid. u is the wave velocity.

Wave velocity
The waves in the beach erosion problem are propagated on shallow water beaches. The linear wave theory should not be applied directly. Because the wave is severely non-linear in shallow water, it is a broken wave. Therefore, a simplified formula[2] for the velocity of the breaking wave is given.

$$u = \left(\frac{H_b}{2}\right)\sqrt{\frac{g}{h}} \tag{3}$$

$$H_b = 0.39 g^{0.2} (T H_0{}^2)^{0.4} \tag{4}$$

where H_b is the height of the breaking wave, T is the wave period,and h is the beach water depth.

Sandcastle deformation rate
We define the ratio of eroded sand volume to the initial total volume as the sandcastle deformation rate, in *d*. If *d* reaches *30%*, the sandcastle is considered severely deformed.

$$d = \frac{\iint \Delta E \, d_x \, dy}{V_s} = \frac{\iint K\,(\tau_w - \tau_c)^a \, d_x \, dy}{V_s} \times 100\% \tag{5}$$

where V_s is the initial volume of the sandcastle.

3.2 Several Typical 3D Geometric Model Examples and Results

Based on the above models, we have tested different shapes and orientations of sandcastle models to explore their resistance to wave erosion. In this paper, three kinds of basic 3D geometric shapes which are the same volume, triangular pyramid, hemisphere and water-drop type, are selected to display and analyze. And the results are compared as follows.

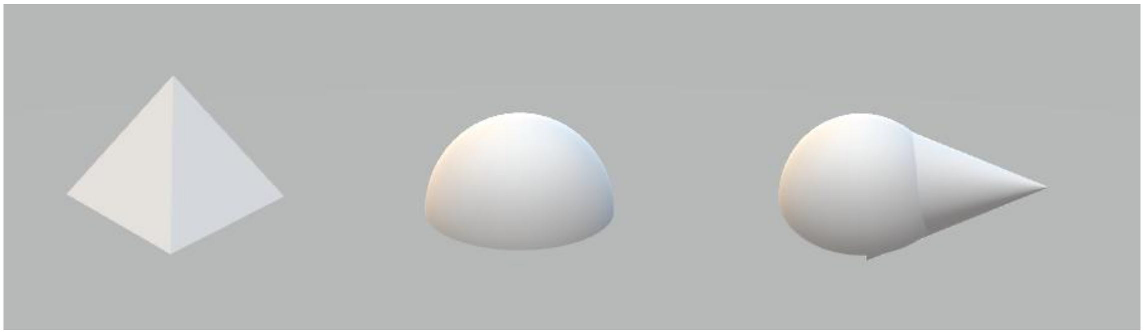

Figure 1: Several typical 3D geometric model

3.2.1 Triangular Pyramid Sandcastle Model

The first sandcastle model we considered is a triangular pyramid. Its top view is a regular triangle. We compared the cases where the corner and side of the triangle faced the wave respectively. We use waves of the same velocity.

Top View Model

From the analysis of the top view, when the corner of the triangular pyramid faces the waves, the overall pressure is less.The stress and wave changes are shown in the figure below.

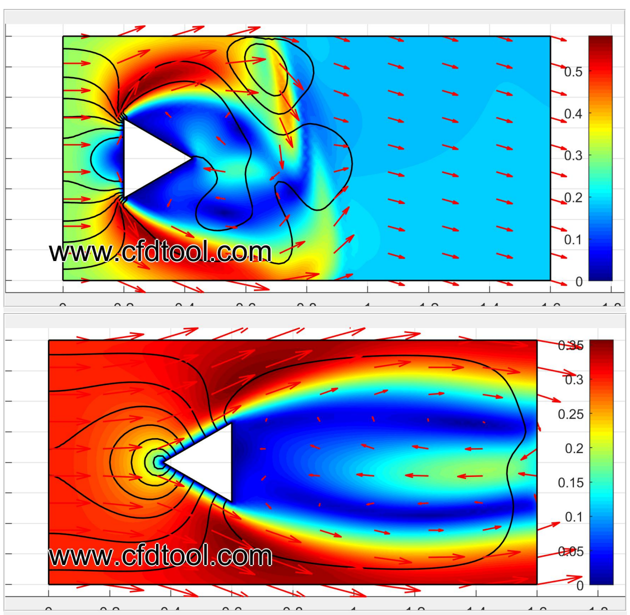

Figure 2: Top-view triangular pyramid model (The website watermark is provided by the free version of the CFD tool platform. This image is our independent simulation result, but not from network.)

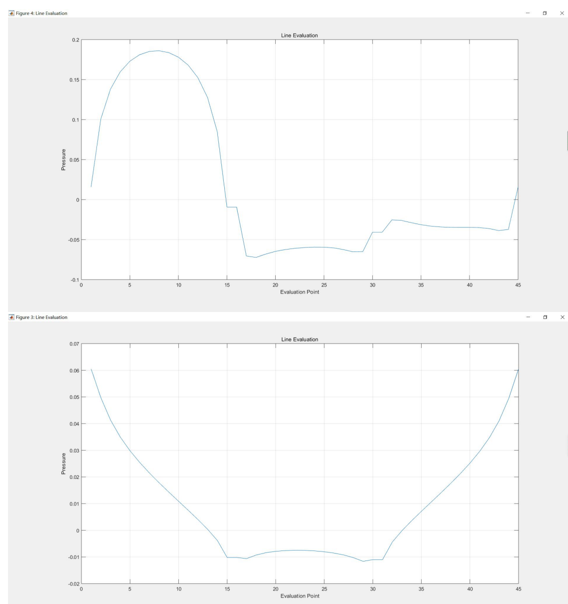

Figure 3: Edge pressure curve

Side View Model

The side view of the model is an inclined plane. Based on the MATLAB, we simulate the change of the outer contour during the wave washing process. We simulate the change of the outer contour during the wave washing process.

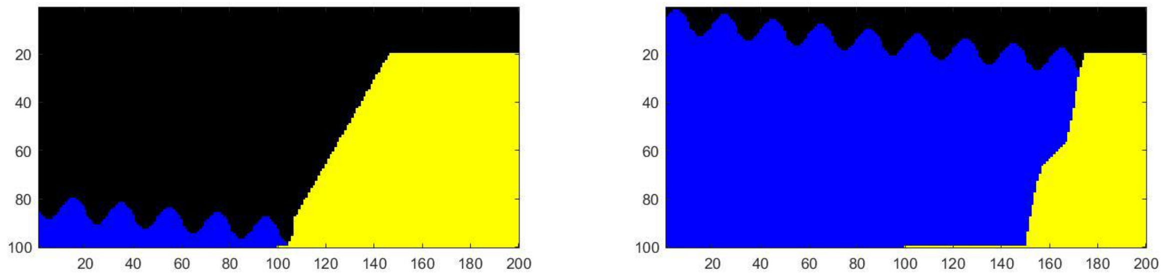

Figure 4: Front and rear side view of triangular pyramid sandcastle model seawater erosion

3.2.2 Hemispherical Sandcastle Model

Top View Model

Considering our daily life, sand castles often appear as nearly spherical pits after being eroded by water. Sphere seems to be in a better state of stability. At the same time, the model needs a more stable base. So we simulate the characteristics of the hemisphere. From a top view, its shape is circular.

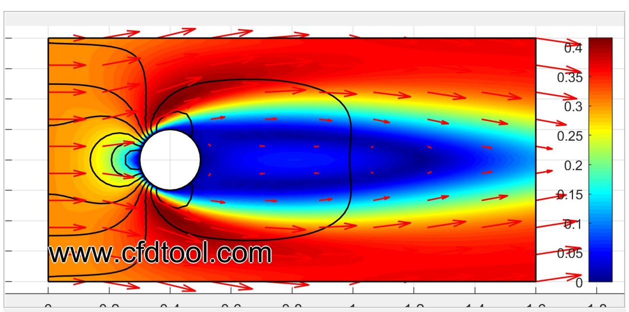

Figure 5: Top-view hemispherical model

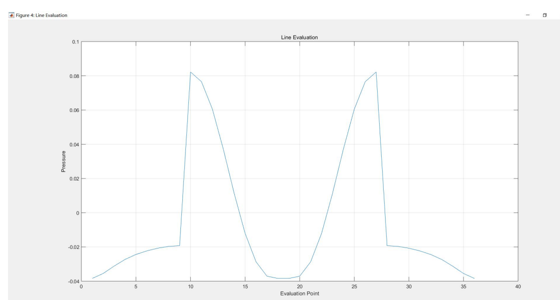

Figure 6: Edge pressure curve

Side View Model

The side view of the model is also semi-circular, and the side is affected by seawater erosion as shown in the figure.

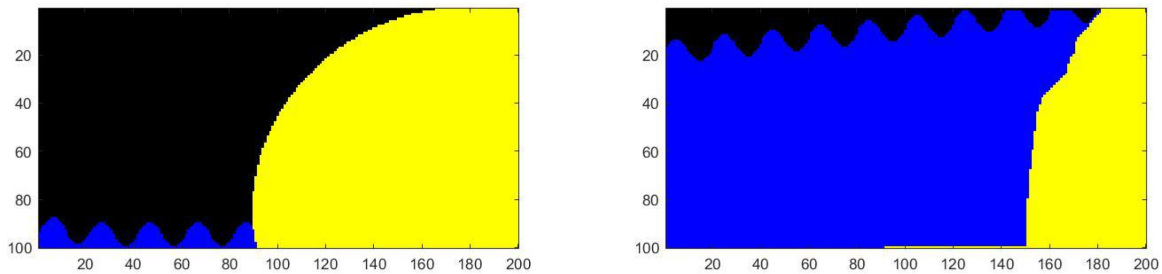

Figure 7: Front and rear side view of Hemispherical Sandcastle model seawater erosion

3.2.3 Water Drop Type Sandcastle Model

In connection with nature, streamline is recognized as the shape with the least resistance to the movement process. The shapes of fish and raindrops are all related to streamline. We consider a kind of sand castle model with water drop shape. The side of the sand castle is approximately streamline.

To simplify, we use a quarter sphere and a half cone to simulate the shape of water drop.

Top View Model

From a top view, its shape is a combination of two streamlines. Besides, we compare two different orientations of this model. The results show that the impact of the pointed head towards the waves is less.

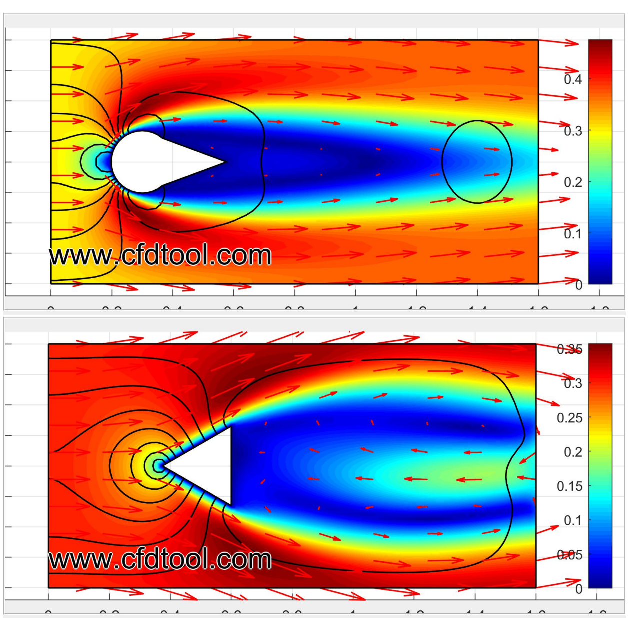

Figure 8: Top-view hemispherical model

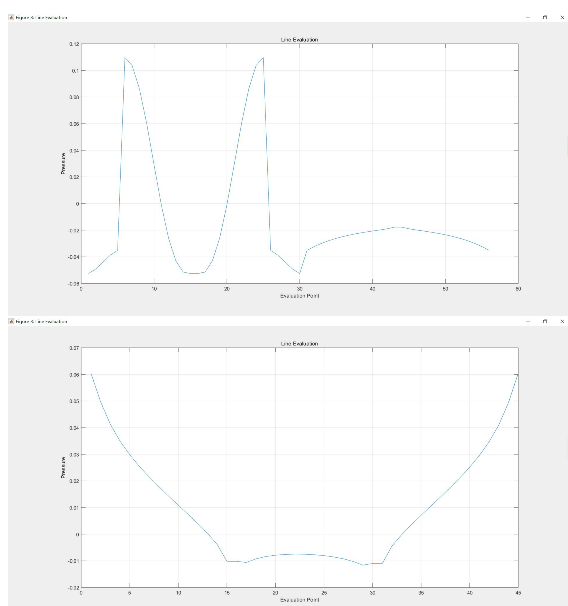

Figure 9: Edge pressure curve

Side View Model
Since our water drop model is a combination of a quarter sphere and a half cone. The side shape is a combination of a quarter arc and a bevel. Therefore, the analysis is similar to the side of the hemisphere and the side of the triangular pyramid.

3.3 Result Analysis

With reference to the data of the National Marine Environment Prediction Center, we set up the approximate wave velocity, wave height, sea temperature and other relevant data. These sandcastle models have the same volume and different shapes. We calculate the time when the volume erosion of Sandburg model reached 50%. We calculate the average forces on the top view and side view of each sand castle model.

Table 1: Average forces on the top view and side view of each sand castle model

Shape	Top View Model	Side View Model
Triangular Pyramid	10.9242	2.7369
Hemisphere	3.4523	1.7025
Water Drop Type	2.9385	2.2406

So the best shape is the Water Drop Type.

4 Problem II: Optimal Sand-to-water Mixture Proportion

A sandcastle can be built on the principle that water forms a liquid bridge between particles, connecting sand particles together. From a microscopic perspective, we analyze the combined forces of liquid bridge forces on sand particles in space under different water contents, and use this to analyze the stability of sandcastles.

To facilitate quantitative calculations, we make the following assumptions: sand diameter $D_{sand}=0.25mm$, sand radius $R=0.125mm$, and surface tension of water is 0.072. On average, each sand touches fourteen sands and seven liquid bridges. We got this result that sand gravity $G_{sand}=2.12\times10^{-10}N$, liquid bridge force $F_{liq}=0.55\times10^{-7}N$. According to the momentum theorem, the impact force of seawater is $F_{water}=2.8274\times10^{-7}N$, so we ignore sand gravity.

Relationship between Liquid Bridge Force and Water Content
We calculate this problem in two steps. In the first step, we calculated the contact radius and contact area between the liquid bridge and the sand under different water contents and distances. The second step is to calculate the liquid bridge force based on the contact area.

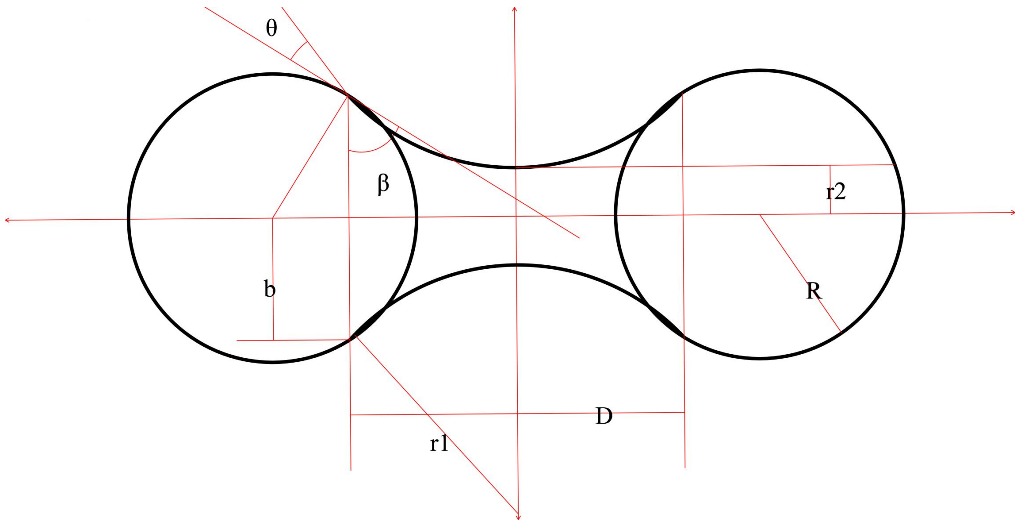

Figure 10: Schematic diagram of fluid bridge between particles

where *D* is the spherical distance, *θ* is wetting angle, r_2 is the radius of the neck of the liquid bridge, r_1 is the outer contour radius of the liquid bridge, and *R* is the particle radius.

Contact Radius and Contact Area

Liquid bridge volume calculation formula is

$$V = \int_0^b 2\pi y H(y) dy = \frac{\pi R}{2}[H^2(b) - D^2] \tag{6}$$

$$H(b) = D + \frac{b^2}{R} \tag{7}$$

where *b* is the radius of the wetting area, that is, the contact radius.The liquid bridge force calculation formula is

$$F_{cap} = \pi \Delta \mathrm{P} r_2^2 + 2\pi r_2 \gamma \tag{8}$$

$$\Delta P = \gamma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \tag{9}$$

where *ΔP* is the capillary pressure and *γ* is the surface tension. According to geometric principles

$$r_1 = \frac{D/2 + R(1 - \cos\beta)}{\cos(\beta + \theta)} \tag{10}$$

$$r_2 = R\sin\beta - [1 - \sin(\beta + \theta)] r_1 \tag{11}$$

then here we have

$$\beta = \arcsin\left(\frac{b}{R}\right) \tag{12}$$

And we think that R>r1>r2, D<<2r1cosθ. We assume that wetting angle θ=1°, then b can be calculated from the water content and the spherical distance.

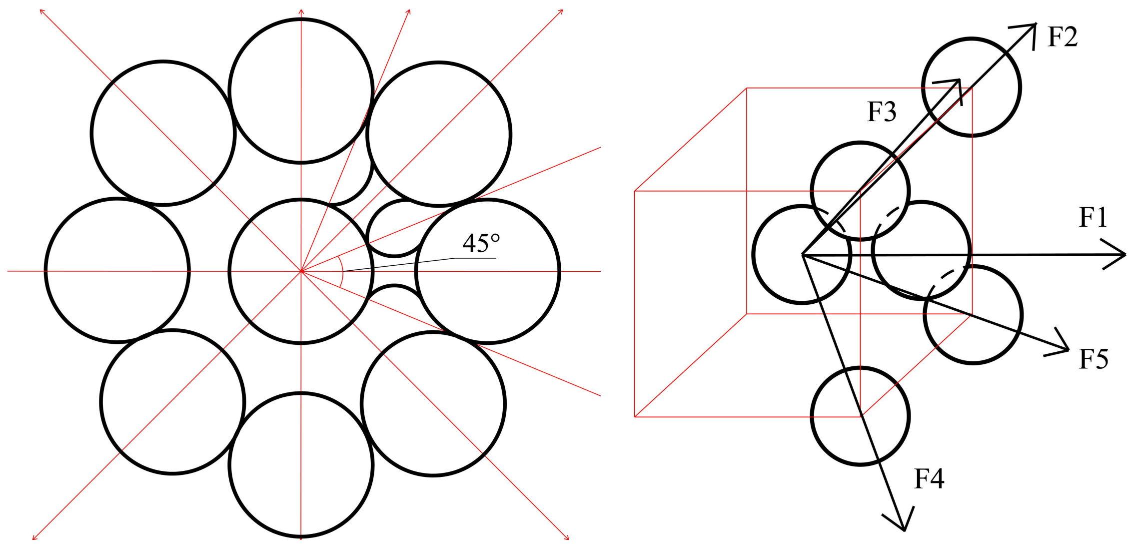

Figure 11: The largest contact area between liquid bridge and sand

If the liquid bridge continues to increase, the liquid bridge force will decrease sharply during fracture.

Fluid Bridge Force

The coarse-grained sample is simply dispersed into spherical particles, and the water form is regarded as the liquid bridge. The liquid bridge force Fliq is considered to be the result of the capillary repulsion force generated by the matrix suction Fψ and the capillary gravity caused by the surface tension F_σ[4].

$$F_{liq} = F^{\sigma} + F^{\psi} = 2\pi(r_2R)\sigma - \pi(r_2R)^2\psi \tag{13}$$

$$\psi = \sigma(\frac{1}{r_1R} - \frac{1}{r_2R}) \tag{14}$$

When $F_\sigma > F_\psi$, the liquid bridge force is positive and the liquid bridge is formed; when $F_\sigma < F_\psi$, the liquid bridge force is negative and the liquid bridge is broken[3].

5 Problem III: Impact of Rain

5.1 Erosion of Raindrops on Sandcastles

We consider the erosion force of raindrops on the sand. First of all, we need to calculate the falling speed of raindrops, and then use the momentum theorem to calculate the force.

Many countries have many different formulas for calculating the drop speed of raindrops. When the diameter of the raindrop is greater than 0.19mm, most poeple use the modified Newton formula to calculate the final raindrop speed:

$$V_m = (17.20 - 0.844d)\sqrt{0.1d}\,, where\ d \geq 0.19mm \tag{15}$$

where d is the diameter of the raindrop, and its general value is 1mm[5].Then, according to the theorem of momentum, we can get that the force of raindrop on sand is:

$$F = \rho\pi R^2 V^2 \tag{16}$$

According to the above formula, we calculate that the erosion force of raindrops is $1.3132 \times 10^{-6} N$, which is greater than the force between the sand calculated in the second question, so the sand can be washed down by raindrops.

5.2 Infiltration of Raindrops on Sandcastles

In addition to the erosion force, rain infiltration will change the water content of the sand, so it is easy to collapse. The permeability is mainly related to the slope of sandcastle. And we get the expression of permeability as follows[6]:

$$f(t) = \begin{cases} R\cos\beta, t \le t_p \\ k_s\left(\cos\beta + \frac{h_s\Delta\theta}{I}\right), t > t_p \end{cases} \tag{17}$$

where $f(t)$ is rainfall infiltration rate, k_s is the saturated permeability coefficient, R is the intensity of the rainfall, t_p is water accumulation time, h_s is the Potential water head of matrix at wet peak, I is the depth of cumulative infiltration, and $\Delta\theta$ is difference between package and initial volume moisture content.

We substitute the parameter value to calculate the result as shown in *Fig.12*. With the increase of the slope angle, the corresponding slope permeability will decrease, but due to the influence of the height on the permeability, the sand castle cannot be increased indefinitely.

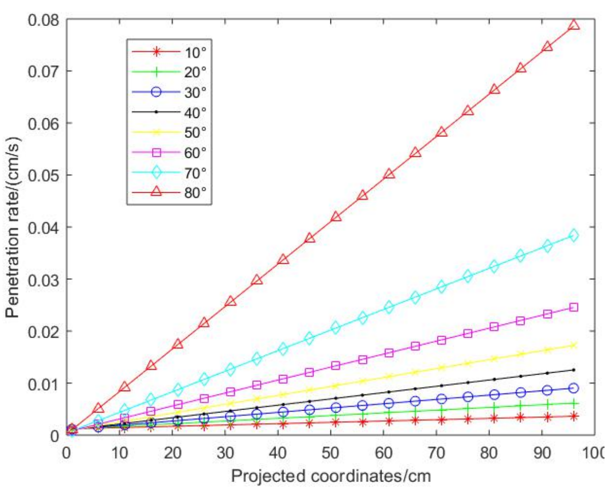

Figure 12: Edge Pressure Curve

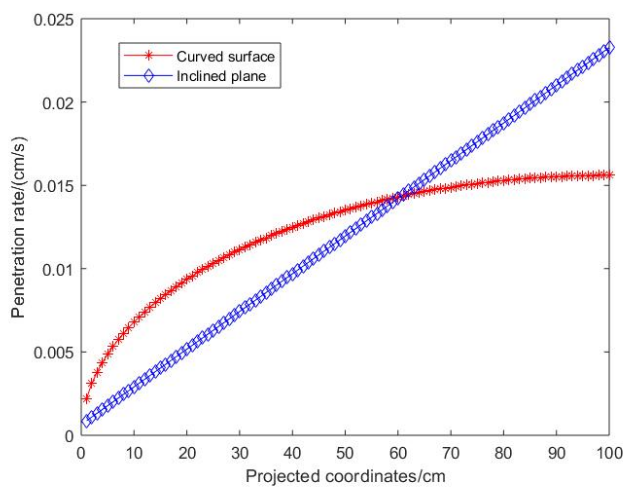

Figure 13: Edge Pressure Curve

Based on the shape of sandcastle, we draw different infiltration rate curves of camber and slope under the same volume in *Figure 13*. It is found that the infiltration rate of the curved surface becomes smaller when the height becomes higher, and the middle is the place with the most sand, so the curved surface is better than the inclined surface under the same volume.

5.3 Modeling and Simulation Based on Cellular Automata

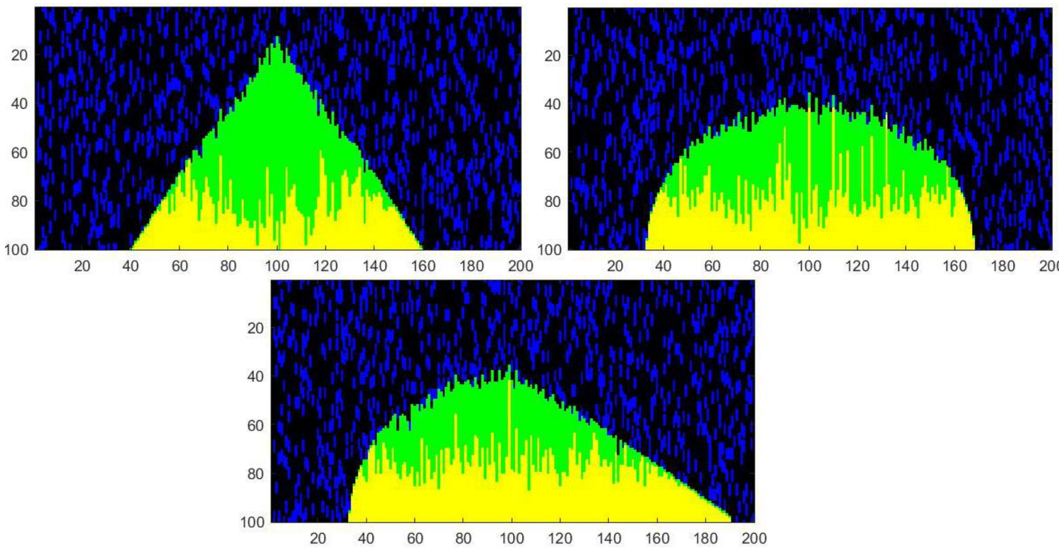

Figure 14: Side view of sandcastle with different shapes eroded by rain

From this we conclude that the spherical sand castle can better resist rain erosion, which is different from the result obtained in the first question.

6 Problem IV: More Strategies

6.1 Build Breakwaters

Many coastal areas have breakwaters to protect the coast. Breakwater is a kind of artificial dam with many holes, which is usually built at a certain distance from the coast. It not only ensures the free access of sea water, but also can effectively disperse wave energy. Therefore, we can build a scattered hard solid in front of the sand castle to simulate the effect of breakwaters.

According to the simulation diagram from the top angle, we found that the part facing the seawater was most impacted. The seawater was blocked and dispersed, forming a space with little pressure behind the sandcastle. From this we propose that adding a baffle in front of the sand castle to resist the impact of the seawater to disperse the seawater can protect the sand castle.

We use different baffle lengths and different distances from the baffle to the sandcastle for simulation and get the following simulation results. The colors in the figure indicate seawater speed.

We take a drop-shaped (round side facing the sea) sandcastle as an example. First we simulated the speed of the seawater around the sandcastle without a baffle *(Figure 15-a)*. Then we have a certain distance between the baffle and the sandcastle. If the baffle is large enough, the seawater can be completely blocked. The speed of the seawater around the sandcastle is 0, and it will not be washed by the seawater *(Figure 15-b)*. If the baffle is reduced to a generally large size, small amount of seawater can bypass the baffle and contact the sand castle, but its speed is still very small.Then close the distance between the baffle and the sandcastle so that they are almost next to each baffle *(Figure 15-c)*. When the baffle is generally large, the seawater is still almost inaccessible to the sand castle *(Figure 15-d)*. When reducing the size of the baffle to make it very small, we found that in the case where the baffle is much smaller than the sand castle, the speed of the seawater around the sand castle is almost the same as without a shelf. *(Figure 15-e)*. Finally, we placed a fence in front of the sand castle. After the seawater passed through the fence, the speed decreased significantly when it reached the sand castle *(Figure 15-f)*.

In summary, we have concluded that adding a breakwater in front of the sandcastle can effectively resist the erosion by seawater. Even fence-type dykes have a significant effect on seawater, but the size must not be too small.

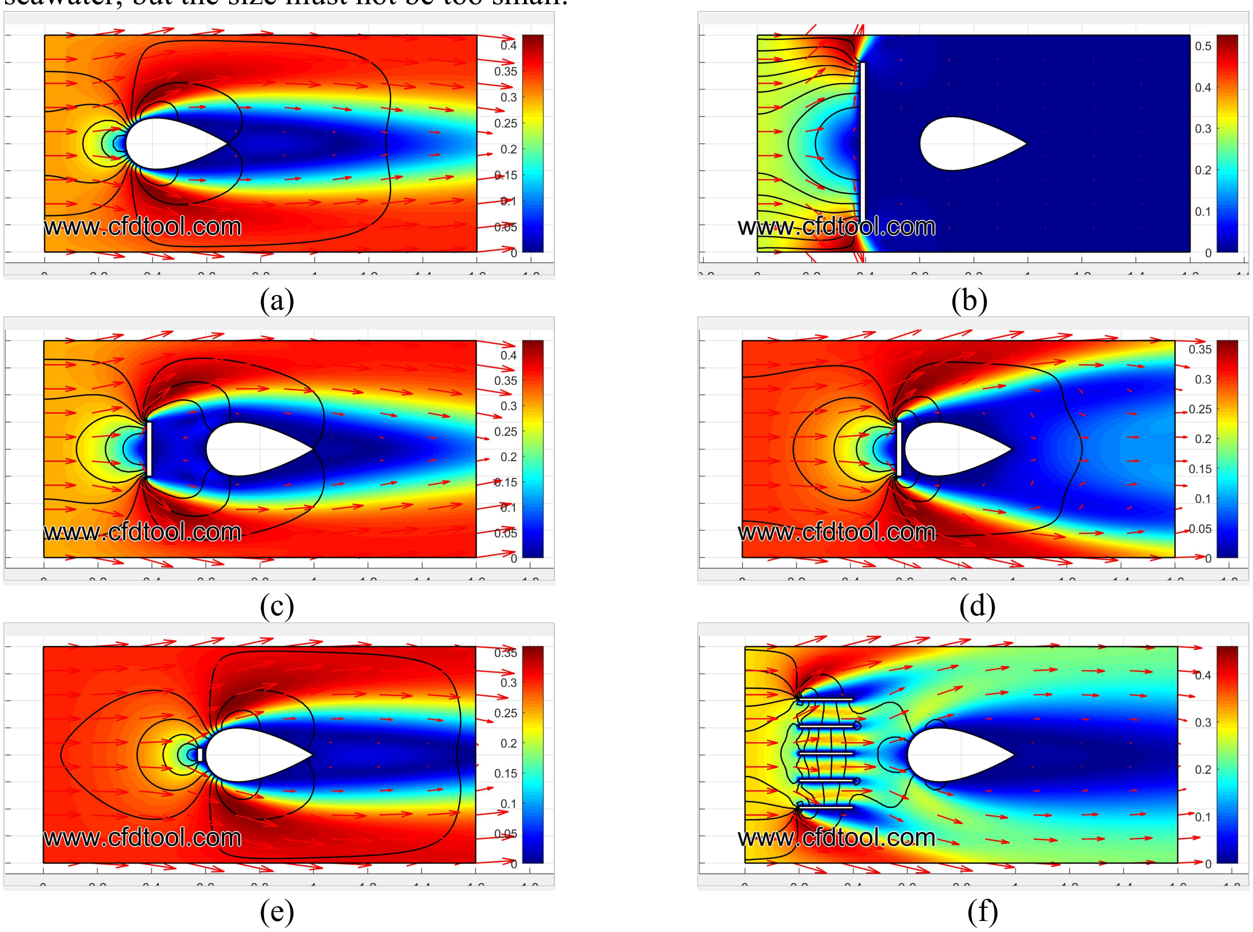

Figure 15: Velocity distribution of seawater near sandcastles behind different baffles

6.2 Mixed With Clay

Clay minerals are malleable when wet with water, can be deformed under relatively low pressure and can remain intact for a long time.When mixed with an appropriate amount of water, a mud ball is formed. Clay has good plasticity and bonding properties. When clay is added to the wet sand, the clay and water combine to form mud balls, and the sand particles are tightly connected to make the sand more resistant. Because the clay will have great strength after drying, when building a sand castle, adding a certain amount of clay requires adding more water, and the sand castle after moderate drying will have very strong hardness.

We compare the erosion speeds of sand castles added with clay and sand castles without clay under the same conditions through simulation, and conclude that clay-bound sandcastles have more sand remaining.

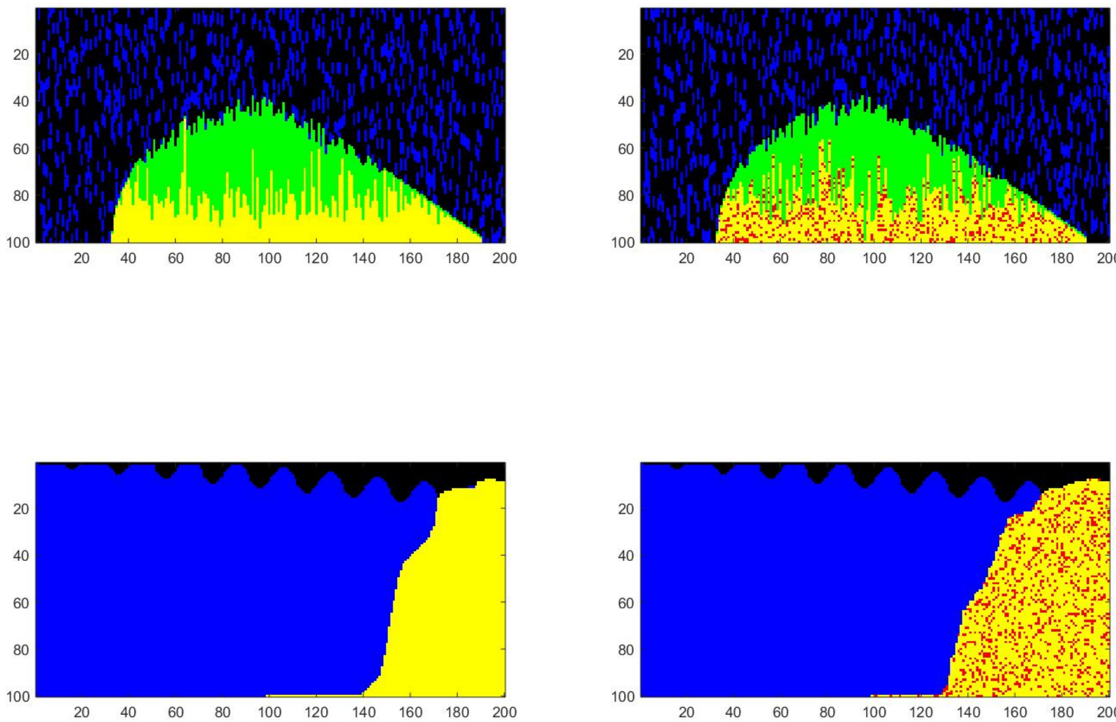

Figure 16: Retention results of different sandcastles after the same length of erosion

By consulting the data, we found that the optimal clay/sand ratio obtained in the experiment was C3S7, that is, 1 unit of clay and 9 units of sand by weight[7]. This proportion of the mixture has the strongest impact resistance, which can resist the impact of seawater for a longer time.

Then we use the program to simulate the time required for a sandcastle to be destroyed(the amount of sand is less than the original 50%) which is made from different materials. The results are shown on *Table2* below, where i is a dimensionless number. Value of i represents the length of time, and the larger the value of i, the longer the time required.

Table 2: Time required for sandcastle to be destroyed and residual sand ratio

	Hemispherical		Triangular Pyramid	
	i	Residual Sand Ratio(%)	*i*	Residual Sand Ratio(%)
Sand	5300	49.98	2900	49.94
Sand(in the rain)	759	49.99	516	49.86
Sand(20% clay contained)	7200	49.89	3580	49.66
Sand(50% clay contained)	8980	49.99	5020	49.83

6.3 Far from the sea

In addition to the beating of sea water, the erosion of the tide is also the main reason for destroying the sand castle. After the high tide, the sandcastle is submerged in the water, and the water content is constantly increasing, and the sand is more easily lost with the seawater. And life experience tells us that the farther away from the beach, the less likely it is to be submerged by high tide. When far enough from the sea, the erosion of sandcastles by seawater can be completely eliminated.

In combination with reality, we believe that the construction of sandcastles on the beach a little farther from the seawater will help the preservation of sandcastles.

7 Sensitivity Analysis

7.1 Sensitivity Analysis for d_s

As we mentioned in problem Ⅱ, the sand diameter d_s is assumed to be *0.25 mm*. When we are analyzing the liquid bridge force between sand and water, the diameter of sand is a more important parameter in the process of stress analysis. In this section, we modify the value of d_s to see if it will affect the result of the model. We set six different values of sand radius from small to large, and substitute them into the second question model to solve the optimal water content of sandcastle. It turns out that these six sizes of sand get the same very similar results. They all reach the maximum liquid bridge force and the most stable state when the sandcastle has a water content of *18%-24%.* So, according to variance analysis, the diameter of sand is not affect the stability of the model.

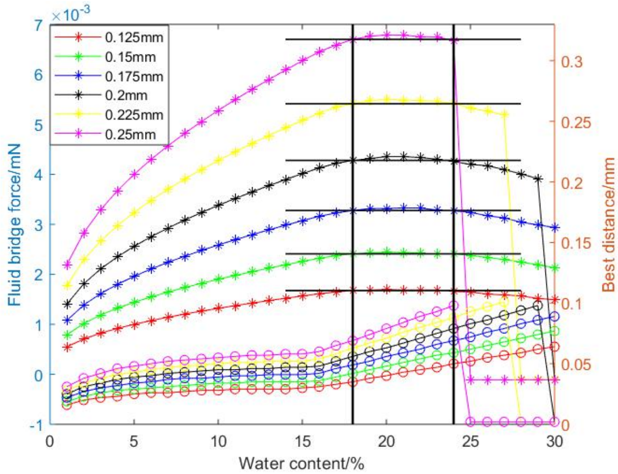

Figure 17: Relationship between maximum liquid bridge force and water content and sand diameter

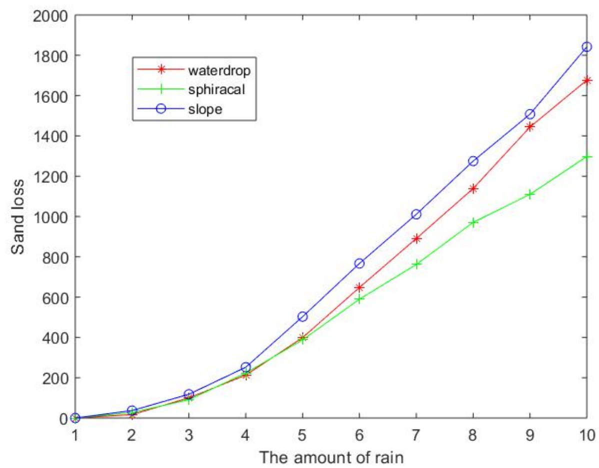

Figure 18: Relationship between rainfall and sand loss

7.2 Sensitivity Analysis for Rainfall

Initially, we assumed a certain amount of rainfall. After analysis, we get the relationship between different rainfall and sand loss, as shown in *Figure 18*.

7.3 Model Robustness Analysis

We build the side view erosion model. Whether the first question is the form of wave erosion or the third question of raindrop erosion, a good simulation effect is achieved. In a word, this model not only vividly simulates the changes of seawater and raindrops in the erosion process, but also achieves the same simulation results on the erosion results of sandcastle models of the same volume and different shapes.

8 Strengths and Weaknesses

8.1 Strengths

We carry out modeling simulation from the top view and the side view, respectively, to make the model simpler. From these two perspectives, the deformation of the outer contour can be clearly analyzed and easily extended to three-dimensional changes.

We consider three typical three-dimensional models in detail, and analyze and compare them from the top and side views. The discussion process is more comprehensive and detailed. In addition, we innovatively proposed a water drop-shaped structure for modeling and analysis.

For each problem, we have built simulation models to facilitate analysis, obtaining and verifying the results.

8.2 Weaknesses

Due to the limited amount of time and data, we simplify the model in some aspects, for example, the erosion of solid materials in seawater are ignored, using the simplified wave velocity formula and so on.

9 Conclusions

We decomposed the three-dimensional sand pile into two projection planes, a top view and a side view, and modeled these two angles respectively. From the top view, the stress distribution of the drop-type sand castle is very smooth as a whole, and the minimum stress can be obtained when the sand castle's head is curved. Unlike arcs, sharp corners face greater resistance to the waves and the stress distribution becomes steeper. But with a proper sharp angle at the tail, smaller eddy currents can be obtained to keep the overall force to a minimum. From the side view, the curved surface has better resistance to the wave's erosion than the inclined plane, because the integral of the shear stress on the sphere is the smallest. So we gave a half-drop-shaped sandcastle basic model, and the simulation results using cellular automata also verified our conclusions.

A model of the relationship between the liquid bridge force and the water content was established, and it was found that the water content of the quartz sand that can reach the most stable conditions is between 18% and 24% .The liquid bridge does not break and overlap under this condition . As we change the size of the sand particles, although the liquid bridge force is constantly changing, the optimal water content remains the same. Therefore, we recommend that the water consumption when making sand castles should be around 20%.

According to the calculation sulution, the order of magnitude of the force rain on sand is close to the order of magnitude of the liquid bridge force between the sand particles, so we divide the effect of raindrops on sand particles into two parts: impact force and infiltration. Using the momentum theorem and the Chen-Young formula for liquid permeation, we established a rain-impact-percolation model and simulated it using a cellular automaton. The simulation results show that the hemispherical sand castle obviously has better erosion resistance than the pyramid sand castle. The anti-erosion ability of the half-drop type sand castle proposed in the first question is somewhere in between.

We also proposed other methods to improve the stability of sand castles, and added fences to eliminate the effect of waves, change the composition of sand, and so on. Through software simulation, we found that the structure of coastal breakwaters has the best effect; using cellular automata model calculations, we found that adding some other ingredients (such as clay) to the sand can significantly improve the stability of the sandcastle.

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